

MatricesIntroduction:

Cayley a French Mathematician; discovered Matrices in the year 1860. Matrices have great Utility in many branches of applied Mathematics; Mechanics and Business.

Definition:

A system of mn numbers arranged in a rectangular formation along m rows and n columns and bounded by the brackets $[]$ is called an m by n matrix; which is written as $m \times n$ matrix. A Matrix is also defined by a single Capital letter.

$$\text{Thus } A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix}$$

is a matrix of order mn . It has m rows and n columns. Each of the mn numbers is called an element of the matrix.

To locate any particular element of a matrix, the elements are denoted by a letter followed by two suffixes which respectively specify the rows and the columns. Thus a_{ij} is the element in the i th row and j -th columns of A . In this notation, the matrix A is denoted by $[a_{ij}]$

Special Types of Matrices:

(1) Row and Column Matrices:

A Matrix having a single row is called a row matrix

e.g.; $[1, 3, 5, 7]$

A matrix having a single column is called a column matrix; e.g.;

$$\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Row and Column matrix (Matrices) are sometimes called Row vectors and Column vectors.

(2) Square Matrix: A matrix having n rows and n columns is called a square matrix of order n .

The determinant having the same elements as the square matrix A is called the determinant of the matrix and is denoted by the symbol $|A|$.

For example; if

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$\text{Then } |A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

The diagonal of this matrix containing the elements 1, 3, 5 is called the leading or; principal diagonal. The sum of the diagonal elements of a square matrix A is called the trace of A .

A square matrix is said to be singular; if its determinant is zero otherwise Non-singular.

(3) Diagonal Matrix : A square matrix all of whose elements except those in the leading diagonal, are zero is called a diagonal matrix.

A diagonal matrix whose all the leading diagonal elements are equal is called a scalar matrix. For example;

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 6 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

are the diagonal and scalar matrices respectively.

(4) Unit Matrix : A diagonal matrix of order n which has unity for all its diagonal ~~element~~ ~~at~~ elements, is called

a Unit matrix of Order n and is denoted by I_n . For example; Unit matrix of Order 3 ~~is~~ is;

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(5) Null Matrix : If all the elements of a Matrix are zero, it is called a Null Matrix or zero matrix and is denoted by ' 0 ' e.g.;

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ is a null Matrix.}$$

(6) Systematic and Skew-Systematic Matrices:

A Square matrix $A = [a_{ij}]$ is said to be systematic when

$a_{ij} = a_{ji}$ for all i and j .

If $a_{ij} = -a_{ji}$ for all i and j so that all the leading diagonal elements are zero, then the matrix is called a Skew-symmetric Matrix. Examples of Symmetric and skew-symmetric matrices are as given;

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \text{ and } \begin{bmatrix} 0 & h & -g \\ -h & 0 & f \\ g & -f & 0 \end{bmatrix}$$

(7) Triangular Matrix :

A Square matrix all of whose elements below the leading diagonal are zero, is called an upper triangular matrix.

(17) Upper triangular matrix :
A square matrix all of whose elements above the leading diagonal are zero, is called lower triangular matrix. Thus

$$\begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & -5 & 4 \end{bmatrix}$$

are upper and lower triangular matrices respectively.

Equality of Matrices :

Two matrices A and B are said to be equal if and only if ;

- (i) They are of the same order;
and (ii) Each element of A is equal to the corresponding element of B.

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